8 Mechanical Properties of Solids

For construction of bridges, considering the mechanical properties of cables used to hold the bridge, thermal expansion of material and stress on the wire is very important. Without considering such elements, the bridge might collapse. These calculations are also made during the construction of dams, buildings and houses. In the real world, the Mechanical properties of Solids are quite significant. Therefore, this chapter introduces a whole new branch of physics.

Topic Notes

- *Elastic Behaviour of Solids*

ELASTIC BEHAVIOUR OF SOLIDS 1

TOPIC 1

ELASTICITY

Elasticity is the quality of an object that allows it to revert to its original shape after a deforming force is removed. For example, if we stretch a rubber band and then release, it snaps back to its original length.

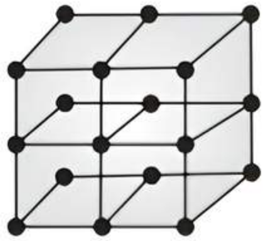
Perfectly elastic bodies are those that promptly and totally restore their original configuration after the deforming force is removed. Quartz fiber is the closest thing to a fully elastic body.

If a body does not regain its original size and shape completely and immediately, after the removal of the deforming force, it is said to be a plastic body and this property is called plasticity.

Perfectly plastic bodies are those that do not revert to their previous shape after removing the deforming force. Putty and paraffin wax has almost completely plastic bodies.

Example 1.1: In the spring-ball model of inter-molecular forces, the balls represent atoms and springs represent interatomic forces. How does this explain the elastic behaviour of the solid?

Ans. The figure of the spring ball model can be drawn as,



The mass of the atom is represented by the spherical or ball-shaped particle, and the interatomic forces between the atoms are represented by the spring, which is denoted here by lines.

Each atom or molecule in a solid is surrounded by other atoms or molecules. Inter-atomic or intermolecular forces bind them together and keep them in a stable equilibrium position. When a material is deformed, the atoms or molecules are moved from their equilibrium positions, causing inter-atomic (or intermolecular) distances to vary. When deforming forces are withdrawn, the interatomic forces appear to drive them back to their original positions. As a result, the body regains its original shape and scale. The process of restoration can be visualised using the spring-ball device model shown in the above figure. Here the balls represent the atoms and the springs represent the inter-atomic powers. Thus, springs represent interatomic forces and balls represent masses.

TOPIC 2

STRESS AND STRAIN

Stress

If an external force deforms a body, an internal force of the reaction is created at each section of the body, which tends to return the body to its original state. Stress is the internal restorative force built up per unit area of the cross-section of the deformed body. Mathematically it is represented as:

$$\text{Stress} = \frac{\text{Applied Force}}{\text{Area}}$$

If there is no permanent change in the configuration of the body, the restoring force is equal and opposite to the external deforming force applied.

Its unit is N/m^2 or Pascal.

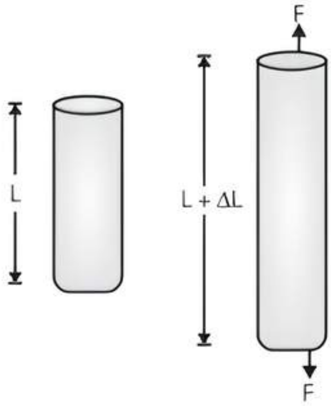
Its dimensional formula is $[ML^{-1}T^{-2}]$.

Classification of Stress:

There are three different types of stress:

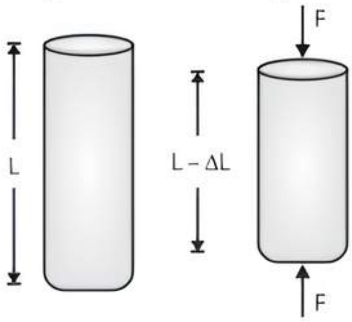
(1) Longitudinal Stress: When a deforming force is applied in a normal direction to the area of cross-section, the stress is referred to as longitudinal stress. It is further categorised in two types

(i) **Tensile stress:** Tensile stress occurs when there is an increase in the length of an object as a result of applied force.



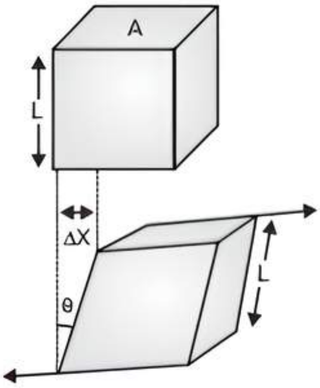
Tensile Stress on a Circular Rod

(ii) Compression stress: Compression stress occurs when there is a decrease in the length of an object as a result of applied force.



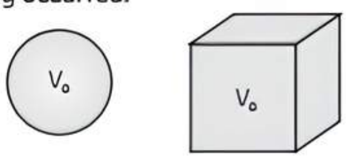
Compressive Stress

(2) Tangential or Shearing Stress: When a deforming force operates tangentially to the surface of a body, the shape of the body changes. Tangential stress is defined as the tangential force exerted per unit area.



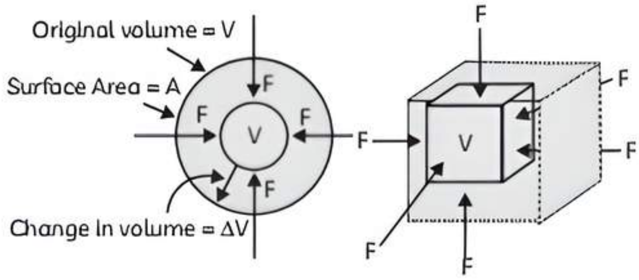
Tangential stress
Deforming force on the surface of a body

(3) Hydraulic or Bulk Stress: When a body is subjected to a uniform force from all sides, the resulting stress is known as hydraulic stress. In case of hydraulic stress, the force F is applied perpendicular to every point on the surface of the body due to which the change in volume of the body occurred.



(Bodies outside the fluid)

Bulk stress



(Bodies Immersed in a fluid)
Hydraulic Stress on Different Surfaces

Strain

When a deforming force acts on a body, the body's shape and size change. Strain is the fractional change in configuration. Mathematically it is written as:

$$\text{Strain} = \frac{\text{Change in configuration}}{\text{Original configuration}}$$

It is unit less and dimensionless quantity as it is a ratio of two same quantities.

According to the change in configuration, the strain is of three following types:

(A) $\text{Longitudinal Strain} = \dfrac{\text{Change in length}}{\text{original length}} = \dfrac{\Delta L}{L}$

(B) $\text{Volumetric Strain} = \dfrac{\text{Change in Volume}}{\text{original Volume}} = \dfrac{\Delta V}{V}$

(C) $\text{Shearing Strain} = \dfrac{\text{Tangential applied force}}{\text{Area of face}} = \dfrac{\Delta X}{L}$

Important

➥ *Materials behave differently under stress. When dropped a glass tumbler shatters into pieces, a rubber ball deforms then bounces back and a metal suffers dents.*

Example 1.2: The maximum load a wire can withstand without breaking when its length is reduced to half of its original length, will:

(a) be double **(b) be half**

(c) be four times **(d) remains the same**

Ans. *(d) remains the same*

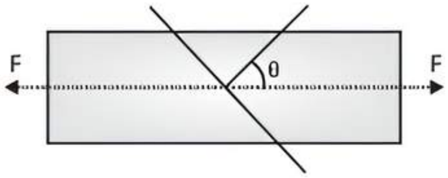
Explanation:

$$\text{Breaking Stress} = \frac{\text{Breaking force}}{\text{Area of cross-section}}$$

By reducing length half its area of cross-section remains the same, and breaking stress does not depend on length. So, the breaking force remains the same.

Example 1.3: Case Based:

The stretching forces acting on the material are referred to as tensile force, which has two components: tensile stress and tensile strain. This *indicates that the material being acted upon is under stress and that the forces are attempting to stretch it. Now consider a bar of cross-section 'A' which is subjected to equal and opposite tensile forces at its ends and a plane section of the bar who's normal makes an angle with the axis of the bar.*



(A) **What is the shearing stress on the given plane?**

(B) **What is the tensile stress on the given plane?**

(C) **When external forces act upon a material, they produce internal stresses within it which cause deformation and produce the maximum stress, now at what angle this will be obtained?**

(a) 0° (b) 30°

(c) 60° (d) 90°

(D) **Assertion (A):** Stress is the internal force per unit area of a body.

Reason (R): Rubber is more elastic than steel.

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true and R is not correct explanation of A.

(c) A is true but R is false.

(d) A is false and R is also false.

(E) **Elasticity has different meanings in physics and in our daily life. Comment.**

Ans. (A) The resolved part of F along the normal is the tensile force on this plane and the resolved part parallel to the plane is the shearing force on this plane.

$$\text{Tensile stress} = \frac{\text{Force}}{\text{Area}} = \frac{F\cos\theta}{A\sec\theta} = \frac{F}{A}\cos^2\theta$$

(Area of cross-section = A sec θ).

(B) Shear stress, the force tending to cause deformation of a material by slippage along a plane or planes parallel to the imposed stress. The resultant shear is of great importance in nature, being intimately related to the downslope movement of earth materials and earthquakes.

$$\text{Shearing stress} = \frac{\text{Force}}{\text{Area}} = \frac{F\cos\theta}{A\operatorname{cosec}\theta} = \frac{F}{2A}\text{Sin}2\theta$$

(C) *(a) 0°*

Explanation: Tensile stress on the plane is maximum when cos2θ is maximum, that is, cos θ = 1 or θ = 0°.

(D) *(c) A is true but R is false.*

Explanation: Stress is an internal force (restoring force) that is created by the object's body. Rubber is less stressed and consequently less elastic since it can be stretched more easily.

(E) In everyday life, a body is said to be elastic if it produces a substantial deformation or strain when subjected to a particular force. In physics, elasticity is the property of a body's substance that prevents it from changing its size or form when stress is applied to it. As a result, if a small strain is produced when applying a particular tension to a body, it will be more elastic.

Example 1.4: **A piece of copper having a rectangular cross-section 15.2 mm × 19.1 mm is pulled in tension with 44,500 N force, producing only elastic deformation. Calculate the resulting strain. Young's modulus of elasticity of copper is 120×10^9 N/m^2.** **[NCERT]**

Ans. Given: $A = 15.2 \times 19.1 \text{ mm}^2$

$= 15.2 \times 19.1 \times 10^{-6} \text{ m}^2$;

$F = 44{,}500$ N

$Y = 120 \times 10^9$ N/m^2

$$\text{Shearing strain} = \frac{F}{AY}$$

$$= \frac{44500}{15.2 \times 19.1 \times 10^{-6} \times 120 \times 10^9}$$

$$= 0.127 \times 10^{-2}$$

Example 1.5: **A steel cable with a radius of 1.5 cm supports a chairlift at a ski area. If the maximum stress is not to exceed 10^8 N/m^{-2}, what is the maximum load the cable can support?** **[NCERT]**

Ans. Maximum load = Maximum Stress × Area of cross-section

$$= 10^8 \times \frac{22}{7}(1.5 \times 10^{-2})^2$$

$$= 7.07 \times 10^4 \text{ N}$$

TOPIC 3

HOOKE'S LAW AND STRESS-STRAIN CURVE

Hooke's Law

Within the elastic limit, Robert Hook discovered that stress is directly proportional to strain.

Thus Stress ∝ Strain

Or Stress = K × Strain

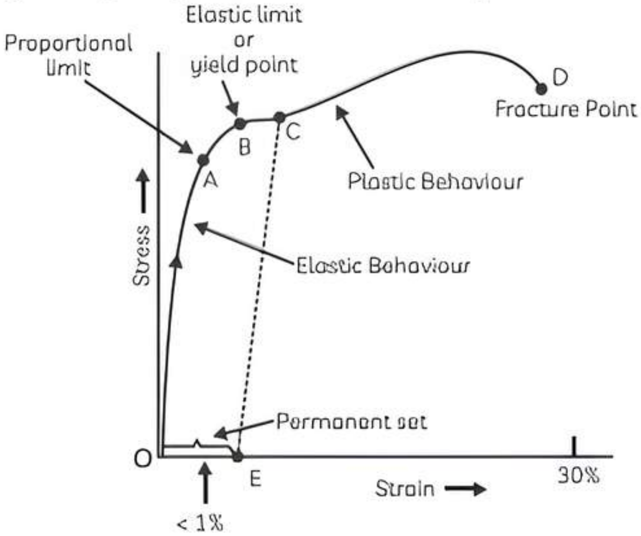
Where K is the constant of proportionality called "Elastic Modulus" of the material. Some materials, like rubber and human muscle, do not obey Hooke's law.

$$E = \frac{\text{Stress}}{\text{Strain}}$$

Here, E is known as modulus of elasticity of the material of the body.

Stress-Strain Curve

The stress-strain curve for a metal wire which is gradually being loaded is shown in the figure:



(1) The graph's starting part OA is a straight line, demonstrating that stress is proportional to strain. Hooke's law is followed up to point A. The proportional limit is denoted by point A. The wire is perfectly springy in this region.

(2) After point A, the tension is not proportional to strain, resulting in a curved region AB. However, removing the load at any point between O and B causes the curve to retrace along BAO and the wire to return to its original length. The graph's part OB is known as the elastic region, and point B is known as the elastic limit or yield point. The yield strength is the stress that corresponds to B.

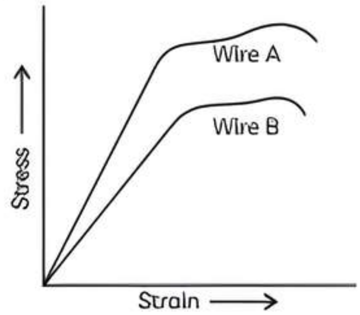
(3) Beyond point B, strain accelerates faster than stress. If the load is removed at any point C, the wire does not return to its previous length, but instead traces the dashed line. Even when the stress is reduced to zero, a residual strain equal to one remains in the wire. It is said that the substance has developed a permanent set. The fact that the stress-strain curve does not retract when the strain is reversed is referred to as elastic hysteresis.

(4) When the load is increased beyond point C, the strain or length of the wire increases dramatically. Constrictions (called necks and waists) form at a few spots along the length of the wire in this region, and the wire eventually breaks at point D, known as the fracture point. Even without any additional stress, the length of the wire increases in the region between B and D. This phase is known as the plastic region and the material is said to go through plastic flow or plastic deformation. The tension corresponding to the breaking point is referred to as the material's ultimate strength or tensile strength.

Important

➥ *During the loading and unloading of a material, the stress-strain curve does not superimpose itself. Some energy is lost as heat during the loading-unloading process. The phenomenon of the stress-strain curve not retracting its path while unloading is called elastic hysteresis.*

Example 1.6: The stress-strain curve for wires of two materials A and B are the shown in the figure. This curve shows which material is more ductile and brittle.



Ans. Material A is more ductile because it has a greater plastic range (a portion of the graph between the elastic limit and breaking point). Material B is more brittle because it has less plastic range.

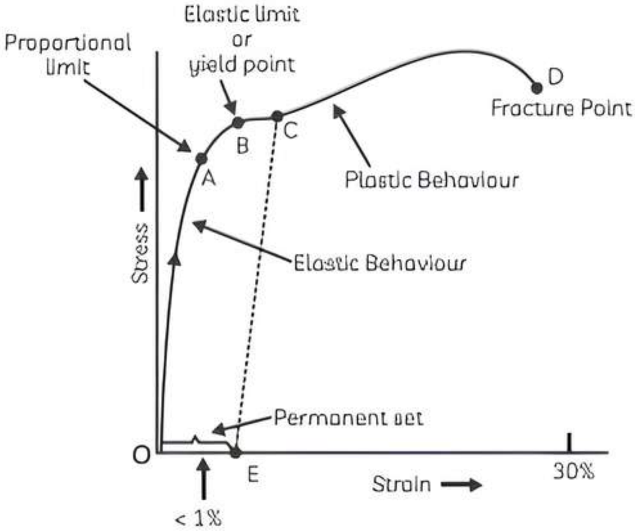
Example 1.7: A hardwire is broken by bending it repeatedly in the opposite direction. Why?

Ans. It is easy to break the wire by bending repeatedly in opposite directions instead of breaking it by stretching directly because by applying alternate force on the wire, the wire loses its strength and hence it is easy to break.

TOPIC 4

ELASTIC MODULUS

The proportional region within the elastic limit of the stress-strain curve (region OA in Figure) is of great importance for structural and manufacturing engineering designs. The ratio of stress and strain, called modulus of elasticity, is found to be a characteristic of the material.



$$\text{Modulus of elasticity} = \frac{\text{Stress}}{\text{Strain}}$$

Types of Modulus of Rigidity

Young's Modulus of rigidity (Y)

Within the elastic limit, it is defined as the normal stress to longitudinal strain ratio. It has the same units as stress since strain lacks a unit. Y is expressed in N/m^2 or Pa. Metals have high Young's modulus values when compared to other materials. In scientific words, the higher the material's Young's modulus, the more elastic it is.

$$Y = \frac{\text{Longitudinal stress}}{\text{Longitudinal strain}}$$

Young's modulus of elasticity is equal to the force required to extend a wire of unit length and unit area of cross-section by a unit amount.

$$Y = \frac{FL}{\pi r^2 \Delta L}$$

Bulk Modulus of Rigidity (*k*)

Within the elastic limit, the ratio of normal stress to the volumetric strain is called the bulk modulus of elasticity. In other words, the ratio of hydraulic stress to the hydraulic strain is called bulk modulus.

$$k = \frac{\text{Normal stress}}{\text{Volumetric strain}}$$

$$= \frac{\frac{-F}{A}}{\frac{\Delta V}{V}} = -\frac{pV}{\Delta V}$$

The SI unit of the bulk modulus is N/m^2.

Important

- *Negative sign shows that the volume decreases with the increase in stress. But for a system in equilibrium, the value of bulk modulus is always positive.*
- *Compressibility of a material is the reciprocal of its bulk modulus of elasticity.*
- *Compressibility (C) = $\frac{1}{k}$. Its SI unit is $N^{-1}m^2$ and CGS unit is $Dyne^{-1}cm^2$.*

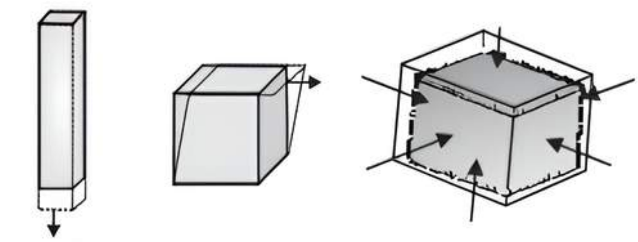
Modulus of rigidity or shear Modulus (η)

Within the elastic limit, the ratio of tangential stress to shear strain is called modulus of rigidity.

$$\text{Strain} = \frac{\text{Change in configuration}}{\text{Original configuration}}$$

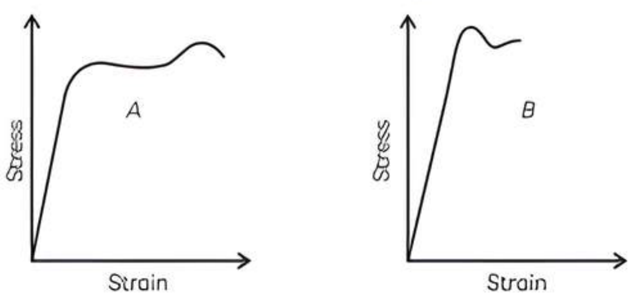
The SI unit of shear modulus is N/m^2.

The shear modulus of a material is always considerably smaller than the Young's modulus for it.



Young's Modulus *Shear Modulus* *Bulk Modulus*

Example 1.8: The stress-strain graphs for materials A and B are shown in Figure.



The graphs are drawn to the same scale.

(A) Which of the materials has the greater Young's modulus?

(B) Which of the two is the stronger material? [NCERT]

Ans. (A) Young's modulus $= \frac{\text{Stress}}{\text{Strain}}$

= slope of stress – strain curve

Here slope is greater for A than B.

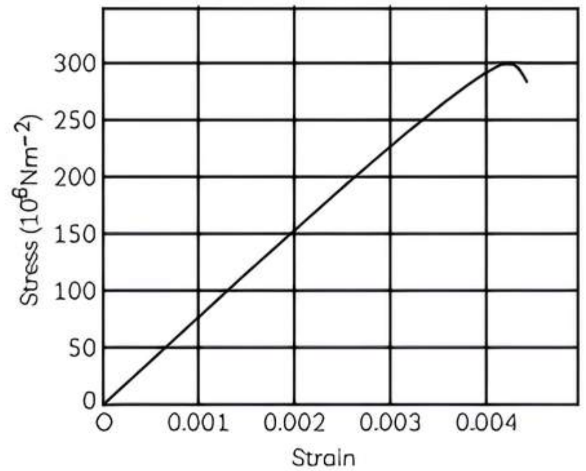
Hence, A has greater Young's modulus.

(B) For stronger material, the stress required for fracture is greater. Hence, A is stronger than B.

Example 1.9: The figure shows the strain-stress curve for a given material. What are

(A) Young's modulus and

(B) approximate yield strength for this material?



[NCERT]

Ans. (A) Young's modulus,

$$= \frac{150 \times 10^6 Nm^{-2}}{0.002}$$

$$= 75 \times 10^9 Nm^{-2}$$

(B) From the graph, yield strength

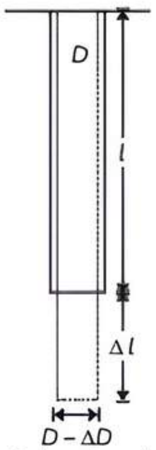
$= 300 \times 10^6 Nm^{-2}$

$= 3 \times 10^8 Nm^{-2}$

TOPIC 5

POISSON'S RATIO

When a deforming force is applied at the free end of a suspended wire of length l and diameter D, then its length increases by Δl but its diameter decreases by ΔD. Now two types of strains are produced by a single force.



Poisson's ratio

(1) Longitudnal strain $= \frac{\Delta l}{l}$

(2) Lateral strain $= \frac{-\Delta D}{D}$

So, Poisson's ratio $= \frac{\text{lateral strain}}{\text{longitudinal strain}} = \frac{\frac{-\Delta D}{D}}{\frac{\Delta l}{l}}$

$$= \frac{-l\Delta D}{D\Delta l}$$

The negative sign shows that longitudinal and lateral strains are in opposite senses.

As Poisson's ratio is the ratio of two strains, it has no units and dimensions. The theoretical value of Poisson's ratio lies between – 1 and 0.5. Its practical value lies between 0 and 0.5.

Factors Affecting Elasticity of Material

(1) **Hammering and rolling:** The crystal grains are broken into small units and the elasticity of the material increases.

(2) **Annealing:** In this process, large crystal grains and the elasticity of the material decrease.

(3) **Presence of impurities:** Depending on the nature of the impurity, the elasticity of the materials increases or decreases.

(4) **Temperature:** Elasticity of most materials decreases with an increase in temperature but the elasticity of incurred steel does not change with a change in temperature.

Elastic Fatigue

Elastic fatigue is defined as the loss in the strength of a material caused due to repeated alternating strains to which the material is subjected.

TOPIC 6

ENERGY STORED IN A DEFORMED BODY

When a wire is stretched, interatomic forces act to counteract the change. Work must be done to counteract these restoring forces. The elastic potential energy of the wire is stored as a result of the work done in extending it.

Elastic potential energy,

$$U = \frac{1}{2} \times \text{Stress} \times \text{Strain} \times \text{Volume of wire}$$

Elastic energy density,

$$\mu = \frac{1}{2} \times \text{Young's modulus} \times (\text{Strain})^2$$

Example 1.10: When the load on a wire is increased slowly from 3 to 5 kgwt., the elongation increases from 0.61 to 1.02 mm. How much work is done during the extension of the wire? Find the value of Young's modulus of the material of the wire, if it is 1 m long and has a cross-sectional area 0.4 mm².

Ans. Work done during the extension of the wire

$$W = \frac{1}{2}(F_1\Delta l_1 - F_2\Delta l_2)$$

$$W = \frac{1}{2}\left(5\times9.8\times1.02\times10^{-3} - 3\times9.8 \times0.61\times10^{-3}\right)$$

$$= 1.61 \times 10^{-2}\ J$$

Young's modulus,

$$Y = \frac{Fl}{A\Delta l}$$

$$= \frac{3\times9.8\times1}{0.4\times10^{-6}\times0.61\times10^{-3}}$$

$$= 1.2 \times 10^{11}\ N/m^2$$

Example 1.11: A steel wire of length 4.7 m and cross-section 3×10^{-5} m² stretches by the same amount as a copper wire of length 3.5 m and cross-section 4×10^{-5} m² under a given load. What is the ratio of Young's modulus of steel to that of copper? [NCERT]

Ans. For steel wire, $A_1 = 3 \times 10^{-5}\ m^2, l_1 = 4.7$ m:

For copper wire, $A_2 = 4 \times 10^{-5}\ m^2, l_2 = 3.5$ m

As $\Delta l_1 = \Delta l_2 = \Delta l$ and $F_1 = F_2 = F$

$$Y_1 = \frac{F_1 l_1}{A_1 \Delta l_1} = \frac{F}{3\times10^{-5}} \times \frac{4.7}{\Delta l}$$

$$Y_2 = \frac{F_2 l_2}{A_2 \Delta l_2} = \frac{F\times3.5}{4\times10^{-5}\Delta l}$$

$$\frac{Y_1}{Y_2} = \frac{4.7\times4\times10^{-5}}{3.5\times3\times10^{-5}} = \frac{18.8}{10.5} \approx 1.8$$

$$\frac{Y_1}{Y_2} = \frac{18}{10} = \frac{9}{5}$$

Example 1.12: The edge of an aluminium cube is 10 cm long. One face of the cube is firmly fixed to a vertical wall. A mass of 100 kg is then attached to the opposite face of the cube. The shear modulus of aluminum is 25 G Pa. What is the vertical deflection of this face? (1 Pa = 1 N/m²) (g = 10 m/s²). [NCERT]

Ans. Given: $A = 0.1 \times 0.1 = 10^{-2}\ m^2$

$F = mg = 100 \times 10$ N;

$$\text{Shearing strain} = \frac{\Delta L}{L} = \frac{\text{Shearing stress}}{\text{Shear Modulus}}$$

$$\frac{\Delta L}{L} = \frac{F/A}{\eta}$$

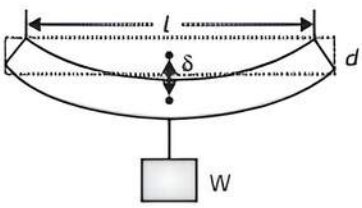
$$\Delta L = \frac{FL}{A\eta} = \frac{100\times10\times0.1}{10^{-2}\times25\times10^9}$$

$$\Delta L = 4\times10^{-7} m$$

TOPIC 7

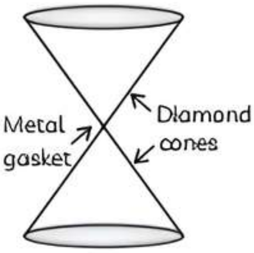
APPLICATIONS OF ELASTIC BEHAVIOUR OF MATERIALS

The elastic nature of materials is critical in everyday life. All engineering designs necessitate a thorough understanding of material elasticity. When developing a structure, for example, the structural design of the columns, beams, and supports necessitates knowledge of the strength of the materials employed. A bridge must be constructed to bear the weight of flowing traffic, wind force, and its weight. Similarly, the use of beams and columns in building architecture is fairly prevalent. The elimination of the problem of beam bending under load is critical in both circumstances. The beam should not bend or break too much. Consider the case of a beam loaded at the center and supported near its ends, as shown in the figure.



A bar of length l, breadth b, and depth d when loaded at the centre by a load W sags by an amount given by,
$$\delta = \frac{Wl^3}{4bd^3Y}$$
Bending can be minimised by utilising a material with a high Young's modulus Y. Depression can be efficiently reduced by increasing the depth d rather than the breadth b. However, a deep bar has the propensity to flex under the weight of passing traffic, thus an I-shaped cross-section is a better choice. This segment has a broad load-bearing surface as well as sufficient depth to prevent bending. This design also minimizes the weight of the beam without reducing its strength, lowering the cost.

Example 1.13: Anvils made of a single crystal of diamond, with the shape as is shown in the given fig. are used to investigate the behaviour of materials under very high pressures. Flat faces at the narrow end of the anvil have a diameter of 0.5 mm, and the wide ends are subjected to a compression force of 50,000 N. What is the pressure at the tip of the anvil?



[NCERT]

Ans. Here, D = 0.5 mm = 0.5×10^{-3} m, F = 50,000 N
$$P = \frac{F}{\pi r^2} = \frac{F}{\frac{\pi D^2}{4}} = \frac{4 \times 50{,}000 \times 7}{22 \times (5 \times 10^{-4})^2} = 2.5 \times 10^{11} \text{ Pa}$$

Example 1.14: A steel ring of radius r and cross-sectional area A is fitted onto a wooden disc of radius R (R > r). If Young's modulus of steel is Y, then find the force with which the steel ring is expanded.

Ans. Initial length (circumference) of the ring = $2\pi r$

Final length (circumference) of the ring = $2\pi R$

Change in length $\Delta L = 2\pi R - 2\pi r$.

$$\text{Now, Young's modulus } Y = \frac{\frac{F}{A}}{\frac{t}{L}} = \frac{\frac{F}{A}}{\frac{R-r}{r}}$$
$$F = AY\left(\frac{R-r}{r}\right)$$

Example 1.15: The Marina Trench is located in the Pacific Ocean, and at one place it is nearly 11 km beneath the surface of the water. The water pressure at the bottom of the trench is about 1.1×10^8 Pa. A steel ball of initial volume 0.32 m^3 is dropped into the ocean and falls to the bottom of the trench. What is the change in the volume of the ball when it reaches the bottom?

(Bulk modulus for steel = 1.6×10^{11} N/m^2.) [NCERT]

Ans. Given, P = 1.1×10^8 Pa, V = 0.32 m^3,

B = 1.6×10^{11} Pa;
$$\Delta V = \frac{PV}{B} = \frac{1.1 \times 10^8 \times 0.32}{1.6 \times 10^{11}} = 2.2 \times 10^{-4} \text{m}^3$$

OBJECTIVE Type Questions

[1 mark]

Multiple Choice Questions

1. Consider a wire whose Length is L and weight is W is connected with weight W_1 suspending from its lower end. Suppose S is the area of the cross-section of the wire. What will be the stress in the wire at a height which is one-fourth of the length of wire from its lower end?

(a) $\frac{W_1}{S}$ (b) $\frac{W_1 + \frac{W}{4}}{S}$

(c) $\frac{W_1 + \frac{3W}{4}}{S}$ (d) $\frac{W_1 + W}{4}$

Ans. *(b)* $\frac{W_1 + \frac{W}{4}}{S}$

Explanation: Total weight at height $\frac{L}{4}$ from its lower end = W_T

W_T = weight suspended+weight of $\frac{L}{4}$ of wire
$$W_T = W_1 + \frac{W}{4}$$
So, stress in wire
$$\text{Stress} = \frac{F}{\text{Area}} = \frac{W_T}{S}$$
$$= \frac{W_1 + \frac{W}{4}}{S}$$

Related Theory

➥ *No material is perfectly elastic or plastic. All the bodies found in nature lie between these limits. When the elastic behaviour of a body decreases, its plastic behaviour increases.*

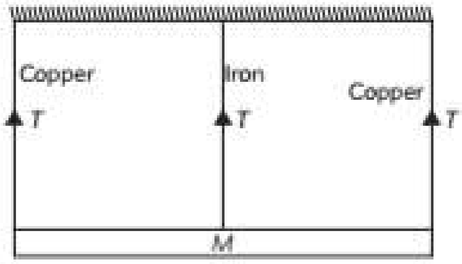
2. **A rigid bar of Mass M is supported symmetrically by three wires each of length L. Those at each end are one of copper and the middle one is of iron. The ratio of their diameters, if each is to have the same tension, is equal to:**

(a) $\frac{Y_{copper}}{Y_{iron}}$ (b) $\sqrt{\frac{Y_{iron}}{Y_{copper}}}$

(c) $\frac{Y^2_{iron}}{Y^2_{copper}}$ (d) $\frac{Y_{iron}}{Y_{copper}}$

[Delhi Gov. QB 2022]

Ans. *(b)* $\sqrt{\frac{Y_{iron}}{Y_{Copper}}}$



The situation is as shown in the above figure. Let T be the tension in each wire. As the bar is supported symmetrically by the three wires, therefore, the extension in each wire is the same

as $$Y = \frac{\frac{F}{A}}{\frac{\Delta L}{L}}$$

If D is the diameter of the wire

Then $$Y = \frac{\frac{F}{\pi\left(\frac{D}{2}\right)^2}}{\frac{\Delta L}{L}} = \frac{4FL}{\pi D^2 \Delta L}$$

As per the conditions of the problem, F(tension), length L, and extension ΔL is the same for each wire

$$\therefore \quad Y \propto \frac{1}{D^2} \quad \text{or} \quad D \propto \sqrt{\frac{1}{Y}}$$

$$\therefore \frac{D_{copper}}{D_{iron}} = \sqrt{\frac{Y_{iron}}{Y_{copper}}}$$

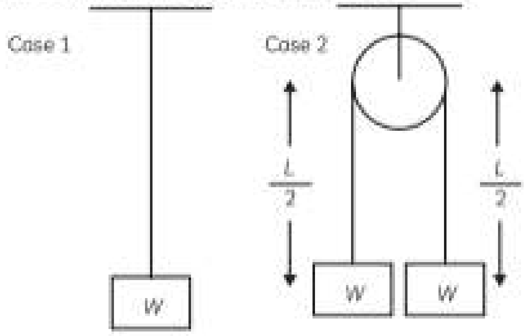
3. **When load W is hung from a wire connected to a roof, it elongates by *l* mm. If the wire is passed through a pulley and two weights W, each is hung at both ends, the wire's elongation will be (in mm):**

(a) zero (b) l

(c) $\frac{l}{4}$ (d) $\frac{l}{2}$

Ans. *(b)* l

Explanation: Consider the length of the wire as L and cross-sectional area A, the material of the wire has Young's modulus as Y.



Then, for 1st case, $$Y = \frac{\frac{W}{A}}{l/L}$$

For 2nd case, $$Y = \frac{\frac{W}{A}}{\frac{2l'}{l}}$$

$$\therefore \quad l' = \frac{l}{2}$$

So, total elongation of both sides = $2l' = l$

Related Theory

➥ *Greater the value of Young's modulus, of a material, greater will be its elasticity. Therefore, steel is more elastic than copper.*

4. **An average value of Poisson's ratio for steels is 0.28, and for aluminium alloys, 0.33 and the volume of materials that have Poisson's ratios less than 0.50 increases under longitudinal tension and decreases under longitudinal compression. What happens to its Poisson's ratio if for any material its Young's modulus increased to 3 times that of its rigidity modulus?**

(a) 1 (b) 0.5

(c) 0.2 (d) 0.8

Ans. *(b)* 0.5

Explanation: Young's modulus,

$$Y = 2\eta = (1+\sigma)$$
$$Y = 3\eta = 2\eta(1+\sigma)$$
$$\sigma = 0.5$$

⚠ Caution

➥ *Students should know that Young's modulus of elasticity and modulus of rigidity only exists for solids and liquids but bulk modulus of elasticity exists for solids, liquids and gases as gas cannot be deformed along one dimension only and cannot undergo shear strain.*

5. Consider a wire made of an alloy of length l and cross-section area A having Young's modulus Y. If a wire is stretched to a certain length l'' some amount of work done occurs if the same wire is again stretched to length l'. What percentage of work done amount is done?

(a) $\frac{YAl'^2}{l}$ (b) $\frac{YAl'^2}{2l}$

(c) $\frac{YAl^2}{l'}$ (d) $\frac{YAl^2}{2l'}$

Ans. *(b)* $\frac{YAl'^2}{2l}$

Explanation: For a wire of length l stretched by a length l''. the restoring elastic force is:

$$F = Y\left[\frac{l''}{l}\right]A$$

The work required to be done against the elastic restoring forces to elongate it further by a length dx is,

$$dW = F.dx = \frac{YA}{l}l'.dl$$

The total work done in stretching the wire from $x = 0$ to x is,

$$W = \int_0^x \frac{YA}{l}l'.dl = \frac{YA}{l}\left[\frac{l'^2}{2}\right]$$

6. According to Hooke's law of elasticity, if stress is increased, the ratio of stress to strain is:

(a) decreased (b) remains the same

(c) increased (d) none of these

[Diksha]

Ans. *(b) remains the same*

Explanation: According to Hooke's law of elasticity, if stress is increased, the ratio of stress to strain remains constant which is additionally referred to as Young's modulus.

$$\text{Stress} \propto \text{Strain}$$

$$\text{Stress} = \text{constant} \times \text{strain}$$

$$\frac{\text{Stress}}{\text{Strain}} = \text{constant}$$

⚠ Caution

➥ *Students must know that Hooke's law is valid only in a linear position of the stress-strain curve. The law is not valid for large values of strain.*

7. Before breaking, a wire can undergo a maximum weight of 20 kg. Now, if the wire is cut into four equal parts. How much maximum weight can each wire undergo?

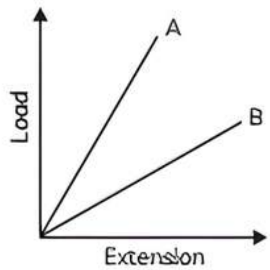
(a) 20 kg (b) 5 kg

(c) 4 kg (d) None of these

Ans. *(a) 20 kg*

Explanation: The stress at which the wire can break or rupture is called breaking stress. This is an intrinsic property of the material and depends only on the nature of the material. Here, the breaking stress of the wire corresponds to the weight of 20 kg mass. As the wire is cut into four equal parts, the nature of the material does not change. Hence, its breaking stress will not change. *i.e.*, each part of the wire can sustain a weight of 20 kg mass.

8. Two wires, A and B, have the same dimensions. Their materials, however, differ. The graphs of their load extension are shown. If Y_A and Y_B are the values of A's and B's Young's modulus of elasticity, respectively, then:



(a) $Y_A > Y_B$ (b) $Y_A < Y_B$

(c) $Y_A = Y_B$ (d) $Y_B = 2Y_A$

Ans. *(a)* $Y_A > Y_B$

Explanation: From the graph.

$$\text{Slope} = \frac{F}{\Delta l}$$

Also, $$Y = \frac{FL}{A\Delta l}$$

$$\text{Slope} = \frac{F}{\Delta l} = Y\frac{A}{L}$$

As more slope, more Y

so, $Y_A > Y_B$.

9. A spring is stretched by applying a load to its free end. The strain produced in the spring is:

(a) volumetric

(b) shear

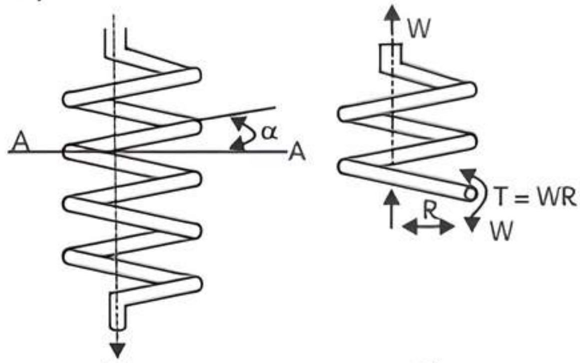
(c) longitudinal and shear

(d) longitudinal

[Delhi Gov. QB 2022]

Ans. *(c) longitudinal and shear.*

Explanation:



(a) (b)

In the case of a bar, the axial force caused by the application of load operates normally to the cross-section, resulting in longitudinal strain. However, in the case of a spring, the force acts along the cross-sectional area of the spring rather than perpendicularly, resulting in torsion and shear strain generation. The spring also expands axially, resulting in longitudinal strain.

10. **Consider a rubber ball and if it is dipped in the pond. Due to the pressure of water from all directions force acts on the ball as a result, the ball seems to be slightly contracted and the fractional change in the object's volume is $\left(\frac{\Delta V}{V}\right)$ and its bulk modulus (B) is related as:**

(a) $\frac{\Delta V}{V} \propto B$ (b) $\frac{\Delta V}{V} \propto \frac{1}{B}$

(c) $\frac{\Delta V}{V} \propto B^2$ (d) $\frac{\Delta V}{V} \propto B^{-1}$

Ans. *(b)* $\frac{\Delta V}{V} \propto \frac{1}{B}$

Explanation: For Volume stress = ΔP

$$\text{Strain} = -\frac{\Delta V}{V}$$

[∵ represent the decrease in volume]

$$\frac{\text{Stress}}{\text{Strain}} = -\frac{\Delta P}{\frac{\Delta V}{V}} = B$$

So, $$\frac{\Delta V}{V} \propto \frac{1}{B}$$

11. **If the wire strain is less than 1/100 and $Y = 6.2 \times 10^{11}$ N/m^2. The wire has a diameter of 1 mm. The maximum weight that can be hung from the wire is:**

(a) 1106 N (b) 1254 N

(c) 4867 N (d) 1687 N

Ans. *(c) 4867 N*

Explanation: $Y = \frac{\text{Stress}}{\text{Strain}}$.

Stress = Y(Strain)

$$\frac{F}{A} = Y \times \text{strain}$$

$$F = Y \times \text{strain} \times A$$

$$= 6.2 \times 10^{11} \times \frac{1}{100} \times \frac{\pi(1\times10^{-3})^2}{4}$$

$$= \frac{6.2 \times \pi \times 10^3}{4} = 4867 \text{ N}$$

12. **The highest stress for which Hooke's law is valid for a given material.**

(a) Ultimate stress

(b) Yield point

(c) Breaking stress

(d) Proportional limit **[Diksha]**

Ans. *(d) Proportional limit*

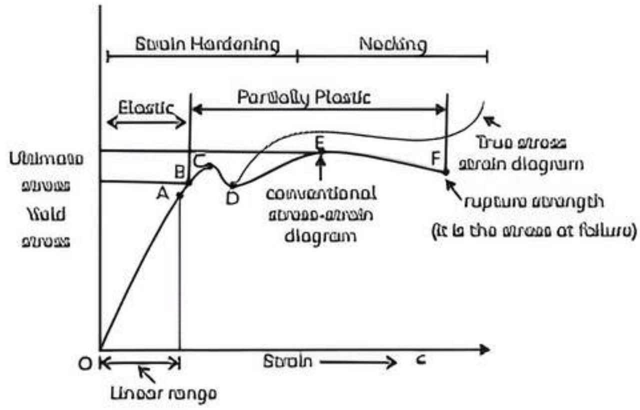
Explanation: Hooke's law holds up to the proportional limit.

Hooke's law in terms of stress and strain is

⇒ Stress ∝ Strain

⇒ $\sigma \propto \varepsilon \Rightarrow \sigma = E\varepsilon$

The constant of proportionality is called the elastic modulus or Young's modules, E. it has the same units as stress. E is a property of the material used.



So, it is evident from the graph that the strain is proportional to stress or elongation is proportional to the load giving a straight line relationship. This law of proportionality is valid up to point A. Point A is known as the limit of proportionality or the proportionality limit.

For a short period beyond point A, the material may still be elastic in the sense that the deformations are completely recovered when the load is removed. The limiting point B is termed as Elastic Limit.

Assertion Reason Questions

Two statements are given one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below:

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true and R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false and R is also false.

13. Assertion (A): Elastic restoring forces may be conservative.

Reason (R): The value of strain for the same stress is different while increasing the load and while decreasing the load.

Ans. *(b) Both A and R are true and R is not the correct explanation of A.*

Explanation: Elastic force is the force formed on a deformed object itself to return to its initial shape. When an elastic material is subjected to an external force, it may undergo deformation till the force is released from the object. Whenever the external force is released, the elastic force within the body of that particular material tends its body to come back to the initial position. Conservative forces are the force which does not depend upon the path, but on the work done. As work done is a point function, the path is not considered in these types of forces.

Related Theory

➥ *Elastic limit is the upper limit of deforming force up to which, if deforming force is removed, the body regains its original form completely beyond which if deforming force is increased, the body loses its property of elasticity.*

14. Assertion (A): A lead is more elastic than rubber.

Reason (R): If the same load is applied on the lead and rubber wire of the same cross-sectional area, the strain of lead is very much less than that of rubber.

[Delhi Gov. QB 2022]

Ans. *(a) Both A and R are true and R is the correct explanation of A.*

Explanation: More elastic the material higher the Young's Modulus.

Young's Modulus of rubber is 0.01-0.1 Gpa whereas for Lead it is 16 Gpa.

Also, $Y = \dfrac{\left(\dfrac{F}{A}\right)}{\dfrac{l}{L}}$

Where

F = Applied Force

A = Cross Section Area

l = Elongation/Decrease in Length

L = Actual Length

Therefore, $\dfrac{l}{L} = \dfrac{F}{AY}$

for F, A same

$\dfrac{l}{L}$ is inversely proportional to Y

So, higher the Young's Modulus, lesser is the strain.

15. Assertion (A): Identical springs of steel and copper are equally stretched. More work will be done on the steel spring.

Reason (R): Steel is more elastic than copper.

Ans. *(a) Both A and R are true and R is the correct explanation of A.*

Explanation: Work done = $\dfrac{1}{2}$ × stress × strain

$$\text{Work done} = \frac{1}{2} \times Y \times (\text{strain})^2$$

As steel is more elastic than copper, more work has to be done in order to stretch the steel.

16. Assertion (A): The bridges were declared unsafe after long use.

Reason (R): Elastic strength of bridges losses with time.

Ans. *(a) Both A and R are true and R is the correct explanation of A.*

Explanation: A bridge during its use undergoes alternating strains a large number of times each day, depending upon the movement of vehicles on it when a bridge is used for a long time, it loses its elastic strength. Due to this, the amount of strain on the bridge for a given stress will become large and ultimately, the bridge may collapse. This may not happen, if the bridges are declared unsafe after long use.

17. Assertion (A): Transverse sound waves do not occur in gases.

Reason (R): Gases cannot sustain shearing strain.

[Delhi Gov. QB 2022]

Ans. *(a) Both A and R are true and R is the correct explanation of A.*

Explanation: Shear strain cannot be tolerated by liquids or gases. Transverse waves, which move in the form of crests and troughs and include a change in shape, cannot thus propagate through a fluid. Because fluids have volume elasticity, compressions and rarefactions that entail volume changes can propagate through them.

18. Assertion (A): Two identical solid balls, one of ivory and the other of wetclay, are dropped from the same height on the floor. Both balls will rise to the same height after bouncing.

Reason (R): Ivory and wetclay have the same elasticity.

Ans. *(d) A is false and R is also false.*

Explanation: Ivory is more elastic than wetclay. Hence, the ball of ivory will rise to a greater height. In fact, the ball of wetclay will not rise at all, it will be somewhat flattened permanently.

19. Assertion (A): Work is required to be done to stretch a wire. This work is stored in the wire in the form of elastic potential energy

Reason (R): Work is required to be done against the intermolecular forces of attraction.

Ans. *(a) Both A and R are true and R is the correct explanation of A.*

Explanation: When we exert tensile stress on a wire, it will get stretched and the work done in stretching the wire will be equal and opposite to the work done by interatomic restoring force. This work is stored in the wire in the form of elastic potential energy. Whereas work done can be derived as $W = \int F.dl$. Where F is a force applied on the wire and dl is changed in length.

CASE BASED Questions (CBQs)

[**4** & **5** marks]

Read the following passages and answer the questions that follow:

20. *Young's modulus is a measure of a solid's stiffness or resistance to elastic deformation under load. It relates stress to strain along an axis or line. The basic principle is that a material undergoes elastic deformation when it is compressed or extended, returning to its original shape when the load is removed. More deformation occurs in a flexible material compared to that of a stiff material. The Young's modulus of a wire is a measure of its stiffness and is defined as the ratio of stress to strain.*

(A) If there are two wires of the same material and the same length while the diameter of the second wire is two times the diameter of the first wire, then what will be the ratio of extension produced in the wires by applying the same load?

(B) The Young's modulus of a wire of length L and radius *r* is Y. If the length is reduced to half, what will be its Young's modulus?

(C) If we can easily stretch rubber and most elastic bands are made of rubber then why can't we use rubber for the manufacturing of springs?

Ans. (A) Both wires are of the same materials, so both will have the same Young's modulus, and let it be Y.

$$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{F}{A\frac{\Delta L}{L}}$$

F = applied force

A = area of cross-section of a wire

Now, $Y_1 = Y_2$

$$\frac{FL}{(A_1)(\Delta L_1)} = \frac{FL}{(A_2)(\Delta L_2)}$$

Since, load and length are the same for both.

$$r_1^2 \Delta L_1 = r_2^2 \Delta L_2$$

$$\left(\frac{\Delta L_1}{\Delta L_2}\right) = \left(\frac{r_2}{r_1}\right)^2 = 4$$

$$\Delta L_1 : \Delta L_2 = 4:1$$

(B) Young's modulus will remain the same, as it is constant for a particular material and doesn't depend on the physical dimension of the wire.

(C) A spring needs a large restoring force when it is deformed, which in turn depends upon the elasticity of the material of the spring. Since, Young's modulus of elasticity of steel is more than that of rubber, steel is preferred in making the springs.

21. *Most of us would have seen a crane used for lifting and moving heavy loads. The crane has a thick metallic rope. The maximum load that can be lifted by the rope must be specified. This maximum load under any circumstances should not exceed the elastic limit of the material of the rope. By knowing this elastic limit and the extension per unit length of the material, the area of cross-section of the wire can be evaluated. From this, the radius of the wire can be calculated.*

(A) The work done when a wire of length l and area of cross-section A is made of material of young's modulus Y is stretched by an amount x is:

(a) $\frac{YAx^2}{L}$ (b) $\frac{YAx^2}{2L}$

(c) $\frac{YAx}{2L}$ (d) $\frac{YA}{2L}$

(B) The Young's modulus of steel is 2.0×10^{11} w/m^2. If the interatomic spacing for the metal is 2.8×10^{-10} m, find the increase in the interatomic spacing for a force of 10^9 N/m^2.

(a) 0.014 Å (b) 0.020 Å

(c) 0.025 Å (d) 0.030 Å

(C) Assertion (A): The shape and size of rigid body remain unaffected under the effect of external forces

Reason (R): The distance between two particles remains constant in rigid body.

(a) Both A and R are true and R is the correct explanation of A.

(a) Both A and R are true and R is not correct explanation of A.

(c) A is true but R is false.

(d) A is false and R is also false.

(D) A wire can support a load Mg without breaking. If it is cut into two equal parts then each wire can support the load of:

(a) $\frac{Mg}{4}$ (b) $\frac{Mg}{2}$

(c) Mg (d) 2Mg

(E) Two wires are made of the same metal. The length of the first wire is half that of the second wire and its diameter is double that of the second wire. If equal loads are applied on both wires, the ratio of increase in their lengths is:

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$

(c) $\frac{1}{10}$ (d) $\frac{1}{8}$

Ans. (A) *(b)* $\frac{YAx^2}{2L}$

Explanation: Young's Modulus

$$Y = \frac{\text{Normal Stress}}{\text{Longitudnal Strain}} = \frac{\frac{F}{A}}{\frac{l}{L}}$$

F = Force

A = Area

l = change in length

L = original Length

x = change in Length (Given)

$$\text{Average extension} = \frac{0+x}{2} = \frac{x}{2}$$

Now, Work Done = Force.

$$\text{Average extension} Y = \frac{FL}{Al}$$

$$F = \frac{YAl}{L}$$

$$\text{Work done} = \frac{YAl}{L} \cdot \frac{x}{2}$$

$$= \frac{YAx}{L} \cdot \frac{x}{2}$$

[$l = x$ (given)]

$$\text{Work done} = \frac{YAx^2}{2L}$$

(B) *(a) 0.014 Å*

Explanation: Given, $Y = 2.0 \times 10^{11}$ N/m^2

$L = 2.8 \times 10^{-10}$ m

F = force

A = Area

Δl = charge in length

$$\frac{F}{A} = \frac{10^9 N}{m^2}$$

Force constant, $K = \frac{F}{\Delta l}$

So, Y = Modulus of elasticity

$$= \frac{F \times l}{A \times \Delta l}$$

Or $$\Delta l = \frac{F \times l}{A \times \Delta l}$$

$$\Delta l = \frac{10^9 \times 2.8 \times 10^{-10}}{2 \times 10^{11}}$$

$$= 1.4 \times 10^{-12} m$$

Or $\Delta l = 0.014$ Å

(C) *(a) Both A and R are true R is the correct explanation of A.*

Explanation: Any entity in which the separation between any two particles is constant is referred to as a rigid body.

Hence, the size and shape of the body stay the same even in the presence of external forces due to the constant spacing between all particles.

A soft substance, however, may alter in size and shape when subjected to external pressures.

(D) *(c) Mg*

Explanation: Maximum load supported by the cable is directly proportional to the breaking stress.

Since,

Breaking stress = $\frac{F}{A}$

Where, F is the force and A is the cross-sectional area.

As we see that the breaking stress is independent of the length of the cable. So, if the cable is cut in two equal parts, the maximum load that can be supported by either part of the cable remains the same as before.

(E) (d) $\frac{1}{8}$

Explanation:

$$Y = \frac{F}{\pi(2r)^2} \times \frac{\frac{L}{2}}{\Delta L_1}$$

$$= \frac{F}{\pi r^2} \times \frac{L}{\Delta L_2}$$

$$= \frac{\Delta L_1}{\Delta L_2} = \frac{1}{8}$$

VERY SHORT ANSWER Type Questions (VSA)

[1 mark]

22. As temperature increases, the strain of the body increases, as a result, the stiffness of the material is reduced, which causes a decrease in the magnitude of the modulus of elasticity, why does this happen?

Ans. When the temperature is increased, the atomic grip gets loosened. Due to this, the material is stretched disproportional to the applied stress. In other words, strain increases with the increase in temperature and hence the value of modulus of elasticity decreases.

23. Young's modulus for steel is much more than that for rubber. For the same longitudinal strain, which one will have greater tensile stress? [NCERT Exemplar]

Ans. Young's modulus, $Y = \frac{\text{Stress}}{\text{Strain}}$

and according to the question strain is equal.

$$Y \propto \text{Stress}$$

$$\frac{Y_{Steel}}{Y_{rubber}} = \frac{(\text{Stress})_{Steel}}{(\text{Stress})_{rubber}}.$$

As the $Y_{steel} > Y_{rubber}$

$$\frac{Y_{Steel}}{Y_{rubber}} > 1$$

$(\text{Stress})_{Steel}$ is larger than $(\text{Stress})_{rubber}$.

24. After doing heavy exercises and weight lifting, Rahul started experiencing elastic fatigue. What is elastic fatigue?

[Mod. Delhi Gov. QB 2022]

Ans. It is a property of an elastic body due to which it becomes less elastic as it has been subjected to repetitive deforming forces alternatively.

Related Theory

➥ *Elastic fatigue, like physical exhaustion in humans, may be compared. If a person has been exhausted for a few days, having one day off may relieve his stress and tiredness, allowing him to resume productivity; similarly, if a body is not subjected to deforming pressures, it can be relieved of elastic fatigue.*

25. Consider two wires A and B of the same material. We will subject wire A to repetitive deforming force for several days and leave wire B untouched. Now, after several days if we give similar vibrations to both the wires, it is observed that the vibrations in wire A would die sooner and wire B would vibrate for a longer time. Wire B is observed to have more elasticity and wire A has undergone elastic fatigue. Why does this situation take place? [Diksha]

Ans. Elastic fatigue happens when a body is subjected to repeated strains or deforming forces even within elastic limits. Under such circumstances, a body loses its property of elasticity. It involves elastic exertion due to repetitive moments.

26. A metal rod of length L and cross-sectional area A, is rigidly fixed between two walls. The Young's modulus of its material is Y and the coefficient of linear expansion is α. The rod is heated so that its temperature is increased from 0°C to θ°C. Find the force exerted at the ends of the rod.

Ans. From the definition of Young's modulus

$$Y = \frac{FL}{A\Delta l}$$

$$F = \frac{YA\Delta l}{l}.$$

Also, $\Delta l = l \alpha \theta$

$$F = \frac{YAl\alpha\theta}{l} = YA\alpha\theta$$

27. Why do spring balances show wrong readings after they have been used for a long time? **[Delhi Gov. QB 2022]**

Ans. When a spring balance is used for an extended period of time, it develops elastic fatigue; the spring of such a balance takes longer to return to its original shape and so does not provide an accurate measurement.

Caution

➥ *Students must remember that if a body is deformed beyond its elastic limit, it will become permanently deformed. The elastic limit is the utmost extent to which a deforming force may be applied without permanently altering a body's shape.*

28. Unlike automotive tires, large aircraft tires are typically filled with nitrogen. Natural rubber is a better material for maintaining pressure for nitrogen-filled tires and it's used to avoid jerks as well. What makes this possible?

Ans. Padding of vulcanized rubber having a large area of hysteresis loop is used in shock absorbers between the vibrating system and the flat board. As the rubber is compressed and released during each vibration, it dissipates a large amount of vibrational energy, a mechanical property of fluids.

SHORT ANSWER Type-I Questions (SA-I)

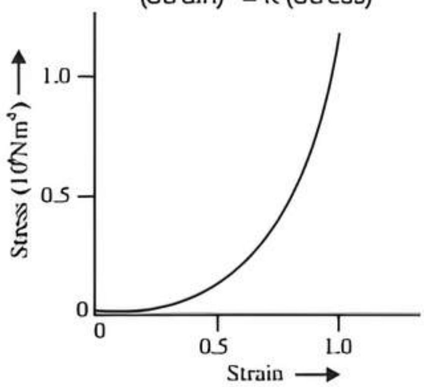
[2 marks]

29. Draw stress-strain curve for elastomers (elastic tissue of the Aorta). **[Delhi Gov. QB 2022]**

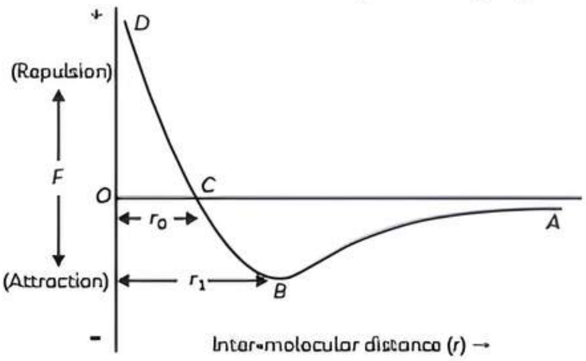
Ans. It is given that stress is proportional to the square of the strain for the elastic tissue of the aorta.

$\therefore$ $\quad$ Stress $\propto$ (Strain)^2

or $\quad$ (Strain)^2 = K (Stress)



30. In the diagram, a graph between the intermolecular force F acting between the molecules of a solid and the distance r between them is shown. Explain the graph.



Ans. As the intermolecular distances r decreases, the force of attraction between the molecules increases.

When the distance decreases to r_1 the force of attraction is maximum.

As the distance further decreases, the attractive force goes on decreasing and when the distance decreases to r_0, the force becomes zero. When the distance decreases below r_0, the molecules begin to repel and the repulsive force increases rapidly.

31. A wire of length l area of cross-section A and Young's modulus Y is stretched by an amount x. What is the work done? **[Delhi Gov. QB 2022]**

Ans. Restoring force in extension,

$$x = F$$

$$x = \frac{AY_x}{L}$$

Work done in stretching it by $dx = dW = F.dx$

Work done in stretching it from zero to $x = W$

$$W = \int dW = \int_0^x F dx$$

$$W = \int_0^x \frac{AY}{L} x\, dx = \frac{1}{2} \frac{AYx^2}{L}$$

32. Two wires of different materials are suspended from a rigid support. They have the same length and diameter and carry the same load at their free end.

(A) Will the stress and strain be the same in each wire?

(B) Will the extension in both wires be the same?

Ans. (A) Stress in both the wires is the same as both the wires have the same diameter and carry the same load at their free end. Strain will be different if two wires are of different materials, even if the stress is the same.

(B) Because the original length of two wires is equal to the strain produced in them different, hence extensions in the wires will not be the same.

33. Suppose two steel and copper springs are stretched in the same way and a person tries to stretch the string but feels that in stretching both springs different amounts of work is done. Which tasks will necessitate additional effort?

Ans. Young's modulus of steel is greater than that of copper. In order to produce the same extension, a larger force will have to be applied on the steel spring than that of the copper spring. Hence, more work will be done on the steel spring.

34. A steel rod ($Y = 2 \times 10^{11}$ $N.m^{-2}$; and $\alpha = 10^{-5}$ C^{-1}) of length 1 m and area of cross-section 1 cm^2 is heated from 0°C to 200°C, without being allowed to extend or bend. What is the tension produced in the rod? [NCERT Exemplar]

Ans. Here, $L_0 = 1m$; $A = 1\ cm^2$,

$\Delta L = 2 \times 10^{-3} m$

$Y = 2 \times 10^{11} N\ m^{-2}$

Change in temperature $\Delta t = 200°C$

$$L_T = L_0(1 + \alpha \Delta t)$$

$$L_T - L_0 = L_0 \alpha \Delta t)$$

$$\Delta L = 1 \times 10^{-5} \times 200 = 2 \times 10^{-3}$$

$$Y = \frac{FL_0}{A\Delta L}$$

$$F = \frac{YA\Delta L}{L_0}$$

$$F = \frac{2\times10^{11}\times10^{-4}\times2\times10^{-3}}{1}$$

$$= 4\times10^{4} N$$

Tension produced in the rod is 4×10^4 N.

SHORT ANSWER Type-II Questions (SA-II)

[**3** marks]

35. Two identical solid balls, one of ivory and the other of wet clay, are dropped from the same height on the floor. Which one will rise to a greater height after striking the floor, and why? [Diksha]

Ans. As the ivory ball is more elastic than the wet-clay ball, it will tend to retain its shape instantaneously after the collision. Hence, there will be a large energy and momentum transfer compared to the wet clay ball. Thus, the ivory ball will rise higher after the collision.

36. A truck is pulling a car out of a ditch using a steel cable that is 9.1 m long and has a radius of 5 mm. When the car just begins to move, the tension in the cable is 800 N. How much has the cable stretched? (Young's modulus for steel is 2×10^{11} Nm^{-2}) [NCERT Exemplar]

Ans. Here, Length of cable $L_0 = 9.1$ m

$r = 5\ mm = 5 \times 10^{-3}$ m, $A = \pi r^2$

Tension in cable,

$F = 800$ N

$Y = 2 \times 10^{11}$ Nm^{-2}

$$\Delta L = \frac{FL}{AY}$$

$$= \frac{800\times9.10}{3.14\times10^{-3}\times10^{-3}\times5\times5\times2\times10^{11}}$$

$\Delta L = 4.64 \times 10^{-5}$m

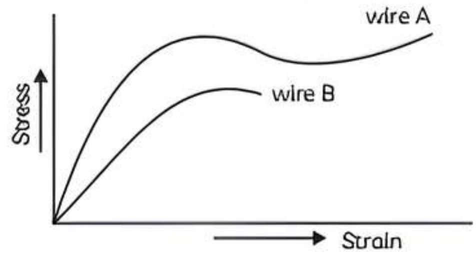
37. The Waterside Trench is located on the West Coast and is nearly eleven kilometers beneath the surface of the water at one point. The water pressure at the trench's bottom is approximately 2.3×10^6 Pa. A steel ball with an initial volume of 0.69 m^3 is dropped into the ocean and falls to the trench's bottom. What is the volume change of the ball when it reaches the bottom? Steel has a bulk modulus of 1.6×10^{11} N/m^2.

Ans. Given, $P = 2.3 \times 10^6$ Pa, $V = 0.69$ m^3, $B = 1.6 \times 10^{11}$ Pa;

Change in volume,

$$\Delta V = \frac{PV}{B} = \frac{2.3\times10^{6}\times0.69}{1.6\times10^{11}} = 0.99\times10^{-5} m^{-3}$$

38. Stress-strain curve for two wires of material A and B are shown in fig.



(A) Which material in more ductile?

(B) Which material has a greater value of Young's modulus?

(C) Which of the two is stronger material?

(D) Which material is more brittle?

[Delhi Gov. QB 2022]

Ans. (A) Wire with the larger plastic region is more ductile material A.

(B) Young's modulus is $\frac{\text{Stress}}{\text{Strain}}$

$\therefore \quad Y_A > Y_B$

(C) For a given strain, larger stress is required for A than that for B.

$\therefore$ A is stronger than B

(D) Material with a smaller plastic region is more brittle, therefore B is more brittle than A.

39. **When we increase the temperature of a material, up to the temperature of 400 K, Young's modulus decreases appreciably. When the temperature rises above 400 K, it decreases at a lower rate and at a very High temperature it is almost constant.**

Ans. Elasticity of a material can be defined as the ability of the material to resist a distorting influence on the material by an external force and to return to its original shape when the external influence is removed. Young's modulus is a mechanical property of solids which gives the stiffness of the material. It can also be defined as the ability of the material to withstand changes in the shape of the material under expansion or compression. Young's modulus of a the material depends on the temperature of the material. When the temperature of the material increase, the atomic vibrations in the crystal structure also increases. This increase in the atomic vibration also increases the atomic distances in the crystal and the atomic force decreases. This decrease in atomic forces leads to a decrease in the Young's modulus of the material.

LONG ANSWER Type Questions (LA)

[4 & 5 marks]

40. **(A) The bulk modulus of rubber is 9×10^8 N/m². To what depth below the surface of the sea should the rubber ball be taken, as to decrease its volume by 0.1%.**

(B) A steel rail is 20 m long and has an area of cross-section 40 sq. cm. Between summer and winter, its length changes by 1 cm. If it is laid in winter, what force parallel to its length is necessary to keep it from increasing the length in the summer? ($Y = 19 \times 10^{10}$ N/m²).

Ans. (A) $\Delta P = B\left(\frac{v}{V}\right)$

Here, $\frac{v}{V} = \frac{0.1}{100}$

$\Delta P = 9 \times 10^5$

Atmospheric pressure at sea level

= 10 m of water column.

Let the ball be taken to a depth h.

Then, $\Delta P = (h - 10)\rho g$

$9 \times 10^5 = (h - 10)(1000)(10)$

$h = 100$ m

(B) $\text{Strain} = \frac{1 \text{ cm}}{20 \times 100\text{cm}} = 0.5 \times 10^{-3}$

$\text{Stress} = Y \times \text{strain}$

$= (19 \times 10^{10})(0.5 \times 10^{-3})$

$= 9.5 \times 10^7$ N/m²

Force = Area of cross-section × stress

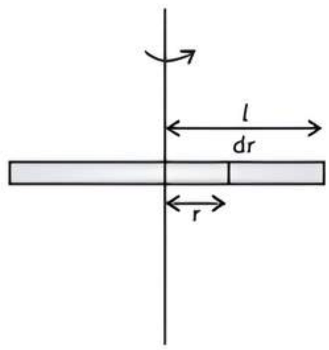
$= (40 \times 10^{-4}) \times (9.5 \times 10^7)$

$= 3.8 \times 10^5$ Newton

41. **A steel rod of length $2l$, cross-sectional area A and mass M is set rotating in a horizontal plane about an axis passing through the center. If Y is Young's modulus for steel, find the extension in the length of the rod. (Assume the rod is uniform).**

[NCERT Exemplar]

Ans. Consider an element at r of width dr.



Let T(r) and T($r + dr$) be the tensions at the two edges.

$$-T(r+dr)+T(r)=\mu\, 2r\, dr$$

Where, $\mu=\frac{\text{mass}}{\text{length}}$

$$-\frac{dT}{dr}dr=\mu\omega^2 r dr$$

At $r = l$, T = 0

$$C=\frac{\mu\omega^2 l^2}{2}$$

$$T(r)=\frac{\mu\omega^2}{2}(l^2-r^2)$$

Let the increase in length of the element dr be

$d(\delta)$

$$Y=\frac{(\frac{\mu\omega^2}{2})\frac{(l^2-r^2)}{2}}{\frac{d(\delta)}{dr}}$$

$$\frac{d(\delta)}{dr}=\frac{1}{YA}\frac{\mu\omega^2}{2}(l^2-r^2)$$

$$d(\delta)=\frac{1}{YA}\frac{\mu\omega^2}{2}(l^2-r^2)dr$$

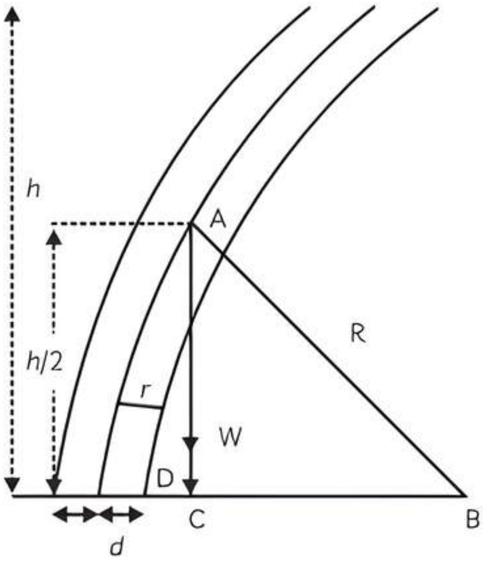
$$\delta=\frac{1}{YA}\frac{\mu\omega^2}{2}\int_0^l(l^2-r^2)dr$$

$$=\frac{1}{YA}\frac{\mu\omega^2}{2}\left[l^3-\frac{l^3}{2}\right]=\frac{1}{3YA}\frac{\mu\omega^2 l^3}{2}$$

The total change in length is

$$2\delta=\frac{2}{3YA}\mu\omega^2 l^3$$

42. **In nature, the failure of structural members usually results from large torque because of twisting or bending rather than due to tensile or compressive strains. This process of structural breakdown is called buckling and in cases of tall cylindrical structures like trees, the torque is caused by its weight bending the structure. Thus, the vertical through the center of gravity does not fall within the base. The elastic torque caused because of this bending about the central axis of the tree is given by $\frac{Ypr4}{4R}$. Y is Young's modulus, r is the radius of the trunk and R is the radius of curvature of the bent surface along the height of the tree containing the center of gravity (the neutral surface). Estimate the critical height of a tree for a given radius of the trunk.** **[Diksha]**



Ans. By Pythagoras theorem: In right-angled ΔABC where point C is just outside the base of the trunk *i.e.*, point C is at D

$$R^2=(R-d)^2+\left(\frac{h}{2}\right)^2$$

$$\because \quad R^2=R^2+d^2-2Rd+\frac{h^2}{4}$$

$\because$ ($d <<< R$ d^2 can be neglected)

$$2Rd=\frac{h^2}{4}$$

Or

$$d=\frac{h^2}{8R}$$

Let the weight of Trunk per unit volume = W_0

The weight of trunk

= Volume × W_0 = $(\pi r^2 h)\, W_0$

Torque by bending the trunk

= Force × perpendicular distance

$$\tau=\pi r^2 h W_0\times d$$

$$\tau=\frac{\pi r^4 Y}{4R}\text{(given)}$$

$$\pi r^2 h W_0\times\frac{h^2}{8R}=\frac{\pi r^4 Y}{4R}$$

$$h^3=\frac{\pi r^4 Y\times 8R}{4R\pi r^2 W_0}=\frac{2r^2Y}{W_0}$$

$$h=\left[\frac{2Y}{W_0}\right]^{1/3} r^{2/3}$$

Hence, h is the critical height given in this expression.

NUMERICAL Type Questions

43. A 4 m long copper wire with a diameter 0.5 mm is used to support a 5 kg weight. Determine the elongation in the wire. Young's modulus for copper is 1×10^{11} N/m^2 and g = 9.8 m/s^2 **(2m)**

Ans. Here, $F = mg = 5 \times 9.8$ N; $l = 4$m.

$$Y = 1 \times 10^{11} \frac{N}{m^2}$$

$$A = \pi r^2 = \frac{\pi d^2}{4} = 3.14 \times \left(\frac{0.5 \times 10^{-3}}{2}\right)^2$$

$$\Delta l = \frac{5 \times 9.8 \times 4}{3.14 \times \left(\frac{0.5 \times 10^{-3}}{2}\right)^2} = 9.99 \times 10^{-4} m$$

44. When a 5 kg weight is hung, the length of a wire increases by 8 mm. What is the increase in length if the conditions remain the same but the radius of the wire is doubled? **(2m)**

Ans. Change in length.

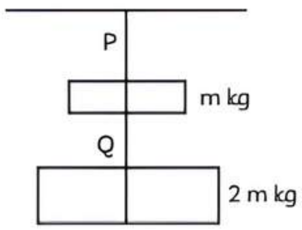
$$\Delta l = \frac{Fl}{AY} = \frac{Fl}{\pi r^2 Y}$$

$$\Delta l \propto \frac{1}{r^2}$$

$$\frac{\Delta l_1}{\Delta l_2} = \frac{r_2^2}{r_1^2}$$

$$\Delta l_2 = \Delta l_1 \times \frac{r_1^2}{r_2^2} = 8 \times \frac{r^2}{(2r)^2} = 2 \text{ mm}$$

45. Two wires P and Q of the same diameter are loaded as shown in the figure. The length of wire P is L meter and its Young's modulus is Y N/m^2 while the length of wire Q is twice that of P and its material has Young's modulus half that of P. Compute the ratio of their elongation.



[Delhi Gov. QB 2022] (2m)

Ans. The ratio of elongation,

$$\Delta l_P = \frac{3mg}{A} \times \frac{L}{Y}$$

$$\Delta l_Q = \frac{2mg}{A} \times \frac{2L}{\frac{Y}{2}} = \frac{8mg}{A} \frac{L}{Y}$$

$$\frac{\Delta l_P}{\Delta l_Q} = \frac{3}{8}$$

46. An aluminium wire 1 m in length and radius 1 mm is loaded with a mass of 40 kg hanging vertically. Young's modulus of Al is 7.0×10^{10} N/m^2. Calculate

(A) tensile stress

(B) change in length

(C) tensile strain and

(D) the force constant of such a wire.

[Delhi Gov. QB 2022] (3m)

Ans. (A) Tensile Stress $= \frac{F}{A} = \frac{mg}{\pi r^2}$

$$= \frac{40 \times 10}{\pi \times (1 \times 10^{-3})^2}$$

$$= 1.27 \times 10^8 \text{ N/m}^2$$

(B)
$$\Delta L = \frac{FL}{AY}$$

$$= \frac{40 \times 10 \times 1}{\pi \times (1 \times 10^{-3})^2 \times 7 \times 10^{10}}$$

$$= 1.8 \times 10^{-3} \text{ m}$$

(C)
$$\text{Strain} = \frac{\Delta L}{L}$$

$$= \frac{1.8 \times 10^{-3}}{1}$$

$$= 1.8 \times 10^{-3}$$

(D)
$$F = Kx = K\Delta L,$$

K = Force constant

$$K = \frac{\Delta F}{L}$$

$$= \frac{40 \times 10}{1.8 \times 10^{-3}}$$

$$= 2.2 \times 10^5 \text{ N/m}$$

47. A 20 kg load is suspended by a metal wire 3 m long with a cross-sectional area of 7 mm^2. Determine the (A) stress, (B) strain, and (C) elongation. The metal has a Young modulus of 3.6×10^{11} Nm^2. **(3m)**

Ans. Force on the wire = mg = 200 N

(A) Stress $(\sigma) = \frac{F}{A} = \frac{200}{7 \times 10^{-6}} = 28.57 \times 10^6$ Pa

(B) Strain $(\varepsilon) = \frac{\sigma}{Y} = \frac{28.57 \times 10^6}{3.6 \times 10^{11}} = 7.93 \times 10^{-5} Pa$

(C) Elongation (Δl) = Strain × Length

$= 85.71 \times 10^6 m$

48. **Compute the bulk modulus of water from the following data: Initial volume = 100.0 liter, Pressure increase = 100.0 atm (1 atm =1.013 × 10^5 Pa), Final volume = 100.5 liter. Compare the bulk modulus of water with that of air (at constant temperature). Explain in simple terms why the ratio is so large. (3m)**

Ans. Initial volume, $V_1 = 100.0 l = 100 \times 10^{-3} m^3$

Final volume, $V_2 = 100.5 l = 100 \times 10^{-3} m^3$

Increase in volume, $\Delta V = V_2 - V_1 = 0.5 \times 10^{-3} m^3$

Increase in pressure,

$\Delta P = 100.0$ atm

$= 100 \times 1.013 \times 10^5$ Pa

Bulk modulus,

$$= \frac{\Delta P}{\frac{\Delta V}{V_1}} = \frac{\Delta P \times V_1}{\Delta V}$$

$$= \frac{100 \times 1.013 \times 10^5 \times 100 \times 10^{-3}}{0.5 \times 10^{-3}}$$

$= 2.026 \times 10^9$ Pa

$$\therefore \frac{\text{Bulk modulus of water}}{\text{Bulk modulus of air}} = \frac{2.026 \times 10^9}{1.0 \times 10^5}$$

$= 2.026 \times 10^4$

This ratio is very high because air is more compressible than water.

49. **A force of 5 × 10^3 N is applied tangentially to the upper face of a cubical block of steel of side 30 cm. Find the displacement of the upper face relative to the lower one, and the angle of shear. The shear modulus of steel is 8.3 × 10^{10} Pa. [Delhi Gov. QB 2022](3m)**

Ans. Area A of the upper face = $(0.30)^2 m^2$

The displacement Δx of the upper face relative to the lower one is given by,

$$\Delta x = \frac{yF}{\eta A},$$

$$\eta = \frac{\frac{F}{A}}{\frac{\Delta x}{y}}$$

$$= \frac{0.30 \times 5 \times 10^3}{8.3 \times 10^{10} \times (0.30)^2}$$

$= 2 \times 10^{-7}$ m

∴ Angle of shear α is given by

$$\tan \alpha = \frac{\Delta x}{y}$$

$$\alpha = \tan^{-1}\left(\frac{\Delta x}{y}\right)$$

$$= \tan^{-1}\left(\frac{2 \times 10^{-7}}{0.30}\right)$$

$= \tan^{-1}(0.67 \times 10^{-6})$